Heat and mass Transfer

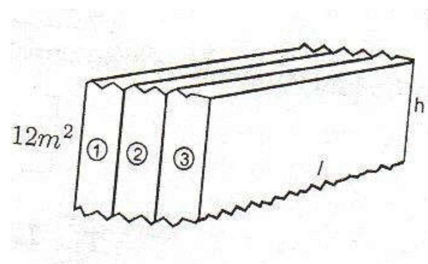
Unit I

Mechanical Engineering BiPSU - 2020-2021 First Semester

1. **Calculate the rate of heat loss through the vertical walls of a boiler furnace of size 4 m by 3 m by 3 m high. The walls are constructed from an inner fire brick wall 25 cm thick of thermal conductivity 0.4 W/mK, a layer of ceramic blanket insulation of thermal conductivity 0.2 W/mK and 8 cm thick, and a steel protective layer of thermal conductivity 55 W/mK and 2 mm thick. The inside temperature of the fire brick layer was measured at 600° C and the temperature of the outside of the insulation 60⁰ C. Also find the interface temperature of layers.**

Given:

Composite Wall



l= 4m b= 3m h= 3m

Area of rectangular wall lb = 4x3 = 12m²

L_1 = 25 cm, k_1 = **0.4 W/mK** } Fire brick

L_2 =0.002m, k_2 = **54 W/mK** } Steel

L_3 = 0.08 m, k_1 = **0.2 W/mK** } insulation

T_1 = 600⁰ C

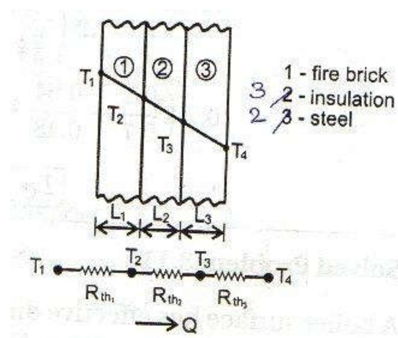
T_2 = 60⁰ C

Find

(i) Q (ii) ($T_3 - T_4$)

Solution

We know that,



$$Q = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$

Here

(ΔT) overall = $T_1 - T_4$

And $\Sigma R_{th} = R_{th1} + R_{th2} + R_{th3}$

$$R_{th1} = \frac{L_1}{k_1 A} = \frac{0.25}{0.4x12} = 0.0521 K/W$$

$$R_{th2} = \frac{L_2}{k_2 A} = \frac{0.08}{0.2x12} = 0.0333 K/W$$

$$R_{th3} = \frac{L_3}{k_3 A} = \frac{0.002}{54x12} = 0.0000031 K/W$$

$$Q = \frac{T_1 - T_4}{R_{th1} + R_{th2} + R_{th3}}$$

$$= \frac{600 - 60}{0.0521 + 0.0000031 + 0.0333}$$

Q = 6320.96 W

(i) To find temperature drop across the steel layer (T_2 - T_3)

$$Q = \frac{T_2 - T_3}{R_{th3}}$$

T_3- T_4 = $Q \times R_{th2}$

= 6320.96× 0.0000031

T_3- T_4 = 0.0196 K .

2. A spherical container of negligible thickness holding a hot fluid at 140^0 and having an outer diameter of 0.4 m is insulated with three layers of each 50 mm thick insulation of k_1 = 0.02: k_2 = 0.06 and k_3 = 0.16 W/mK. (Starting from inside). The outside surface temperature is 30^{0}C. Determine (i) the heat loss, and (ii) Interface temperatures of insulating layers.

Given:

OD = 0.4 m

r_1 = 0.2 m

r_2 = r_1 + thickness of 1st insulation

= 0.2+0.05

r_2 = 0.25m

r_3 = r_2 + thickness of 2nd insulation

= 0.25+0.05

r_3 = 0.3m

r_4 = r_3 + thickness of 3rd insulation

= 0.3+0.05

r_4 = 0.35m

T_{hf} = 140^o C, T_{cf} = 30^o C,

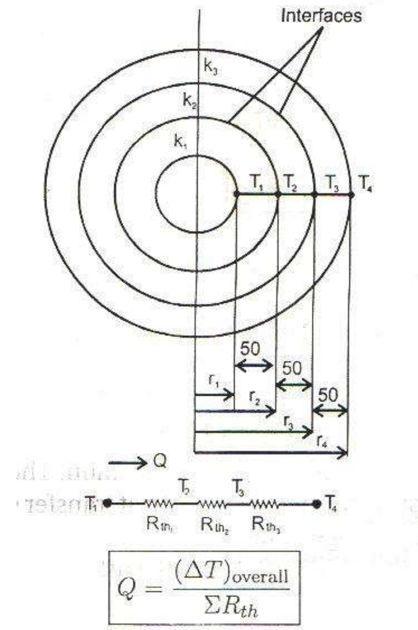
k_1 = 0.02 W/mK

k_2 = 0.06 W/mK

k_3 = 0.16 W/mK.

Find (i) Q (ii) T_2, T_3

Solution



$$Q = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$

$\Delta T = T_{hf} - T_{cf}$

$\Sigma R_{th} = R_{th1} + R_{th2} + R_{th3}$

$$R_{th1} = \frac{r_2 - r_1}{4\pi k_1 r_2 r_1} = \frac{(0.25-0.20)}{4\pi\, x0.02x0.25x0.2} = 3.978^{o}\ C/W$$

$$R_{th2} = \frac{r_3 - r_2}{4\pi k_2 r_3 r_2} = \frac{(0.30-0.25)}{4\pi\, x0.06x0.3x0.25} = 0.8842^{o}\ C/W$$

$$R_{th1} = \frac{r_4 - r_3}{4\pi k_3 r_4 r_3} = \frac{(0.35-0.30)}{4\pi\, x0.16x0.35x0.30} = 0.23684^{o}\ C/W$$

$$Q = \frac{140-30}{0.0796 + 0.8842 + 0.23684}$$

$Q = 21.57$ W

To find interface temperature (T_2 , T_3)

$$Q = \frac{T_2 - T_3}{R_{th1}}$$

$T_2 = T_1 - [Q \text{ x } R_{th1}]$

$= 140 - [91.62\times0.0796]$

$T_2 = 54.17^{0}C$

$$Q = \frac{T_2 - T_3}{R_{th1}}$$

$T_3 = T_2 - [Q \times R_{th2}]$

$= 132.71 - [91.62\times0.8842]$

$T_3 = 35.09^{o}$ C

3. May 2008

A steel tube with 5 cm ID, 7.6 cm OD and k=15W/m ^{o} C is covered with an insulative covering of thickness 2 cm and k 0.2 W/m ^{o}C. A hot gas at 330^{o} C with h = 400 W/m^{2o}C flows inside the tube. The outer surface of the insulation is exposed to cooler air at 30^{o}C with h = 60 W/m^{2o}C. Calculate the heat loss from the tube to the air for 10 m of the tube and the temperature drops resulting from the thermal resistances of the hot gas flow, the steel tube, the insulation layer and the outside air.

Given:

Inner diameter of steel, d_1 = 5 cm =0.05 m

Inner radius,r_1 = 0.025m

Outer diameter of steel, d_2 = 7.6 cm = 0.076m

Outer radius,r_2 = 0.025m

Radius, $r_3 = r_2$ + thickness of insulation

= 0.038+0.02 m

$r_3 = 0.058$ m

Thermal conductivity of steel, $k_1 = 15 W/m^\circ C$

Thermal conductivity of insulation, $k_2 = 0.2 W/m^\circ C$

Hot gas temperature, $T_{hf} = 330^\circ C + 273 = 603 K$

Heat transfer co-efficient at innear side, $h_{hf} = 400 W/m^{2\circ}C$

Ambient air temperature, $T_{cf} = 30^\circ C + 273 = 303 K$

Heat transfer co-efficient at outer side $h_{cf} = 60 W/m^{2\circ}C$.

Length, L = 10 m

To find:

(i) Heat loss (Q)

(ii) Temperature drops $(T_{hf} - T_1)$, $(T_1 - T_2)$, $(T_2 - T_3)$, $(T_3 - T_{cf})$,

Solution:

Heat flow $Q = \frac{\Delta T_{overall}}{\sum R_{th}}$

Where

$\Delta T_{overall} = T_{hf} - T_{cf}$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_{hf}r_1} + \frac{1}{k_1}\ln\left[\frac{r_2}{r_1}\right] + \frac{1}{k_2}\ln\left[\frac{r_3}{r_2}\right] + \frac{1}{k_3}\ln\left[\frac{r_4}{r_3}\right] + \frac{1}{h_{cf}r_4}\right]$$

$$Q = \frac{T_{hf} - T_{cf}}{\frac{1}{2\pi L}\left[\frac{1}{h_{hf}r_1} + \frac{1}{k_1}\ln\left[\frac{r_2}{r_1}\right] + \frac{1}{k_2}\ln\left[\frac{r_3}{r_2}\right] + \frac{1}{h_{cf}r_3}\right]}$$

$$Q = \frac{603 - 303}{\frac{1}{2\pi \times 10}\left[\frac{1}{400 \times 0.025} + \frac{1}{15}\ln\left[\frac{0.038}{0.025}\right] + \frac{1}{0.2}\ln\left[\frac{0.058}{0.038}\right] + \frac{1}{60 \times 0.058}\right]}$$

Q = 7451.72 W

We know that,

$$Q = \frac{T_{hf} - T_1}{R_{th\ conv.}}$$

$$= \frac{T_{hf} - T_1}{\frac{1}{2\pi L} \times \frac{1}{h_{hf}r_1}}$$

$$7451.72 = \frac{T_{hf} - T_1}{\frac{1}{2 \times \pi \times 10} \times \frac{1}{400 \times 0.025}}$$

$$T_{hf} - T_1 = 11.859K$$

$$Q = \frac{T_1 - T_2}{R_{th1}}$$

$$= \frac{T_1 - T_2}{\frac{1}{2\pi L} \times \left[\frac{1}{k_1}\ln\left[\frac{r_2}{r_1}\right]\right]}$$

$$7451.72 = \frac{T_1 - T_2}{\frac{1}{2 \times \pi \times 10} \times \frac{1}{15} \ln \left[\frac{0.038}{0.025}\right]}$$

$$T_1 - T_2 = 3.310\ K$$

$$Q = \frac{T_2 - T_3}{R_{th2}}$$

$$= \frac{T_2 - T_3}{\frac{1}{2\pi L} \times \left[\frac{1}{k_2} \ln \left[\frac{r_3}{r_2}\right]\right]}$$

$$7451.72 = \frac{T_2 - T_3}{\frac{1}{2 \times \pi \times 10} \times \frac{1}{0.2} \ln \left[\frac{0.058}{0.038}\right]}$$

$$T_2 - T_3 = 250.75\ K$$

$$Q = \frac{T_3 - T_{cf}}{R_{th\ conv.}}$$

$$= \frac{T_3 - T_{cf}}{\frac{1}{2\pi L} \times \frac{1}{h_{cf} r_3}}$$

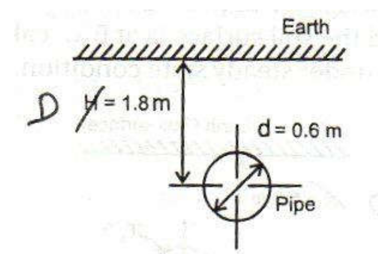
$$7451.72 = \frac{T_3 - T_{cf}}{\frac{1}{2 \times \pi \times 10} \times \left[\frac{1}{60 \times 0.058}\right]}$$

$$T_3 - T_{cf} = 34.07K$$

Nov 2009

4. A long pipe of 0.6 m outside diameter is buried in earth with axis at a depth of 1.8 m. the surface temperature of pipe and earth are 95^0 C and 25^0 C respectively. Calculate the heat loss from the pipe per unit length. The conductivity of earth is 0.51W/mK.

Given



$r = \frac{0.6}{2} = 0.3$ m

L = 1 m

$T_p = 95^o$ C

$T_e = 25^o$ C

D = 1.8 m

k = 0.51W/mK

Find

Heat loss from the pipe (Q/L)

Solution

We know that

$$\frac{Q}{L} = k.S(T_p - T_e)$$

Where S = Conduction shape factor =

$$\frac{2\pi L}{\ln\left(\frac{2D}{r}\right)}$$

$$= \frac{2\pi x\ 1}{\ln\left(\frac{2x\ 1.8}{0.3}\right)}$$

S = 2.528m

$$\frac{Q}{L} = 0.51x2.528(95-25)$$

$$\frac{Q}{L} = 90.25W/m$$

Nov.2010

5. A steam pipe of 10 cm ID and 11 cm OD is covered with an insulating substance k = 1 W/mK. The steam temperature is 200^0 C and ambient temperature is 20^0 C. If the convective heat transfer coefficient between insulating surface and air is 8 W/m^{2}K, find the critical radius of insulation for this value of r_c. Calculate the heat loss per m of pipe and the outer surface temperature. Neglect the resistance of the pipe material.

Given:

$$r_{i=}\frac{ID}{2} = \frac{10}{2} = 5\ cm = 0.05m$$

$$r_{0=}\frac{OD}{2} = \frac{11}{2} = 5.5\ cm = 0.055m$$

k =1 W/mK

$T_i = 200^oC$ $\qquad$ $T_\infty = 20^o$ C

h_0 =8 W/m^{2}K

Find

(i) r_c

(ii) If $r_c = r_o$ then Q/L

(iii) T_o

Solution

To find critical radius of insulation (r_c)

$$r_{0=}\frac{k}{h_0} = \frac{1}{8} = 0.125m$$

When $r_c = r_o$

Kpipe, h_{hf} not given

$$\frac{Q}{L} = \frac{2\pi(T_0 - T_\infty)}{\frac{\ln\left(\frac{r_c}{r_o}\right)}{k} + \frac{1}{h_o r_o}}$$

$$= \frac{2\pi(200-20)}{\frac{\ln\left(\frac{0.125}{0.050}\right)}{1} + \frac{1}{8\,x\,0.125}}$$

$$\frac{Q}{L} = 621\,W/m$$

To Find T_o

$$\frac{Q}{L} = \frac{T_0 - T_\infty}{R_{thconv}}$$

$$T_0 = T_\infty + \frac{Q}{L}(R_{thconv})$$

$$= 20 + 621 \times \left(\frac{1}{8 \times 2\pi \times 0.125}\right)$$

$$T_0 = 118.72^0C$$

November 2011.

6. The temperature at the inner and outer surfaces of a boiler wall made of 20 mm thick steel and covered with an insulating material of 5 mm thickness are 300^0 C and 50^0 C respectively. If the thermal conductivities of steel and insulating material are 58W/m^{0}C and 0.116 W/m^{0}C respectively, determine the rate of flow through the boiler wall.

$L1 = 20 \times 10^{-3}$ m

$k_1 = 58$ W/m^{0}C

$L_2 = 5 \times 10^{-3}$ m

$k_2 = 0.116$ W/m^{0}C

$T_1 = 300^0$ C

$T_2 = 50^0$ C

Find

(i) Q

Solution

$$Q = \frac{(\Delta T)overall}{\Sigma Rth} = \frac{T_1 - T_3}{R_{th1} - R_{th2}}$$

$$R_{th1} = \frac{L1}{k1A} = \frac{0.20 \times 10^{-3}}{58 \times 1} = 3.45 \text{ X } ^{10\text{-}4}\ ^0C/W$$

$$R_{th2} = \frac{L2}{k2A} = \frac{5\text{x } 10^{-3}}{0.116 \times 1} = 0.043\ ^0C/W$$

$$Q = \frac{300 - 50}{3.45 \text{ X } 10{-}4 + 0.043} = 5767.8 \text{ W}$$

Q = 5767.8 W

7. A spherical shaped vessel of 1.2 m diameter is 100 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 200° C. Thermal conductivity of material is 0.3 kJ /mh°C.

Given

$d_1 = 1.2$ m

$r_1 = 0.6$ m

$r_2 = r_1 +$ thick

$= 0.6 + 0.1$

$r_2 = 0.7$ m

$\Delta T = 200^0$C

K = 0.3 kJ /mhr °C = 0.0833 W/m° C

Find

Q

Solution:

$$Q = \frac{\Delta T}{R_{th}} = \frac{T_1 - T_2}{R_{th}}$$

$$R_{th} = \frac{r_2 - r_1}{4\pi r_2 r_1} = \frac{(0.7-0.6)}{4\pi \times 0.0833 \times 0.6 \times 0.7} = 0.2275\ K/W$$

$$Q = \frac{\Delta T}{R_{th}} = \frac{200}{0.2275} = 879.132W$$

November 2011 (old regulation)

8. A steel pipe (K = 45.0 W/m.K) having a 0.05m O.D is covered with a 0.042 m thick layer of magnesia (K = 0.07W/m.K) which in turn covered with a 0.024 m layer of fiberglass insulation (K = 0.048 W/m.K). The pipe wall outside temperature is 370 K and the outer surface temperature of the fiberglass is 305K. What is the interfacial temperature between the magnesia and fiberglass? Also calculate the steady state heat transfer.

Given:

OD = 0.05 m

$d_1 = 0.05$ m

$r_1 = 0.025$ m

$k_1 = 45$ W/mK

$r_2 = r_1 +$ thick of insulation 1

$r_2 = 0.025+0.042$

$r_2 = 0.067$ m

$k_2 = 0.07$ W/mK

$k_3 = 0.048$ W/mK

$r_3 = r_2$ + thick of insulation 2

$= 0.067+0.024$

$r_3 = 0.091$ m

$T_1 = 370$ K

$T_3 = 305$ K

To find

(i) T_2

(ii) Q

Solution

Here thickness of pipe is not given; neglect the thermal resistance of pipe.

$$Q = \frac{(\Delta T)overall}{\Sigma Rth}$$

Here

$$(\Delta T)overall = T_1 - T_3 = 370 - 305 = 65\ K$$

$\Sigma R_{th} = R_{th1} + R_{th2}$

$$R_{th1} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_{2L}} = \frac{\ln\left(\frac{0.067}{0.025}\right)}{2\pi \times 0.07 \times 1} = 2.2414 \text{ K/W}$$

$$R_{th2} = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_{3L}} = \frac{\ln\left(\frac{0.091}{0.067}\right)}{2\pi \times 0.48 \times 1} = 1.0152 \text{ K/W}$$

$$Q = \frac{65}{2.2414+1.0152} = 19.959 \text{ W/m}$$

To find T_2

$$Q = \frac{T_1 - T_2}{R_{th1}}$$

$T_2 = T_1 - [Q \times R_{th1}]$

$= 370 - [19.959 \times 2.2414]$

$T_3 = 325.26$K